

Ground State Laser Cooling Beyond the Lamb–Dicke Limit

G. MORIGI¹, J.I. CIRAC¹, M. LEWENSTEIN², and P. ZOLLER¹

¹ *Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria.*

² *CEA/DRECAM/SPAM, Centre d'Etudes de Saclay, 91191 Gif-sur-Yvette Cedex, France*

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Abstract. – We propose a laser cooling scheme that allows to cool a single atom confined in a harmonic potential to the trap ground state $|0\rangle$. The scheme assumes strong confinement, where the oscillation frequency in the trap is larger than the effective spontaneous decay width, but is not restricted to the Lamb–Dicke limit, *i.e.* the size of the trap ground state can be larger than the optical wavelength. This cooling scheme may be useful in the context of quantum computations with ions and Bose–Einstein condensation.

One of the major goals of atomic physics is to laser cool trapped atoms to the lowest energy state $|0\rangle$ of the confining potential. While this is certainly possible in the so-called Lamb–Dicke Limit (LDL) [1], whereby the spatial dimensions of the ground state $|0\rangle$, a_0 , are much smaller than the wavelength of the cooling laser λ (*i.e.* $\eta = 2\pi a_0/\lambda \ll 1$) [2, 3], there is not known mechanism that achieves this task in the opposite limit ($\eta \geq 1$) [4]. Ground state cooling beyond the LDL is interesting from two perspectives. On the one hand, it is required to perform quantum computations in a linear ion trap [5, 6], as well as to produce some non-classical states of motion of a single ion or atom in scales of the order of the laser wavelength [3]. On the other hand, it may be a way to obtain Bose–Einstein condensation [7, 8] with all-optical means [4, 9].

So far, the most efficient laser cooling mechanism for trapped particles in the LDL is *sideband cooling* [10]. It allows to cool a single particle confined in a harmonic trap practically to the ground state, as first demonstrated experimentally by Diedrich *et al.* [1]. Beyond the LDL, the most effective techniques have been proved to be cooling schemes originally designed to achieve subrecoil temperatures of free atoms, namely, dark state [11] and Raman cooling [12]. The first scheme operates with an angular momentum $J_g = 1 \rightarrow J_e = 1$ internal transition, and it is specially suitable for flat-bottom traps [13], whereas the second one is based on timing laser pulses followed by a repumping process, creating and populating a dark state of zero velocity atoms. In this paper we propose a method which extends the sideband cooling mechanism beyond the LDL, combining ideas of these other schemes. It allows to cool a single

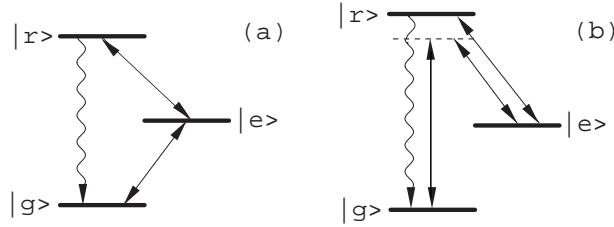


Fig. 1. – Internal configuration of a three level atom, that can be reduced to an effective two level one with ground state $|g\rangle$ and excited state $|e\rangle$. In (a) $|g\rangle \rightarrow |r\rangle$ is an electric dipole allowed transition, whereas $|g\rangle \rightarrow |e\rangle$ and $|e\rangle \rightarrow |r\rangle$ are electric dipole forbidden ones; in (b) $|g\rangle \rightarrow |r\rangle$ $|r\rangle \rightarrow |e\rangle$ are electric dipole allowed transitions. A laser on resonance on the transition $|e\rangle \rightarrow |r\rangle$ pumps optically the atoms into $|g\rangle$.

atom in a harmonic trap to the ground state of the confining potential for $\eta \geq 1$. Although we will present our scheme in a one dimensional situation, it can be easily generalized to higher dimensions, choosing appropriately the geometry of the laser beams; furthermore, it can be easily adapted to the case in which one has many ions in an electromagnetic trap, as in the ion trap quantum computer model [5, 6] by applying it to the cooling of the center-of-mass mode. Note that ground state cooling is a necessary condition to perform quantum gates with ions. In addition, for a set of neutral atoms it operates in the regime in which the heating mechanisms due to reabsorptions can be minimized [14], and therefore it is a promising alternative to evaporative cooling to achieve Bose–Einstein condensation.

Standard sideband cooling of a two-level atom confined in a one-dimensional harmonic trap is achieved by tuning a laser beam to the lower motional sideband of an internal transition (*i.e.* $\delta = -\nu$, where δ is the laser detuning from the two-level resonance, and ν is the trap frequency). Denoting by $|n\rangle$ ($n = 0, 1, \dots$) the eigenstates (Fock states) of the harmonic potential, and by $|g\rangle$ and $|e\rangle$ the two internal atomic states, sideband cooling can be summarized as follows:

(i) The laser induces transitions $|g, n+1\rangle \rightarrow |e, n\rangle$. In this way, if spontaneous emission mainly gives rise to the $|e, n\rangle \rightarrow |g, n\rangle$ transition, in each absorption–emission cycle a quantum of motional energy $\hbar\nu$ is removed. This is precisely the case in the *Lamb–Dicke limit*, since $\eta^2 = \epsilon_R/(\hbar\nu) \ll 1$ (ϵ_R is the photon recoil), and therefore the kinetic energy provided by the emitted photon is not sufficient to excite a vibrational quantum.

(ii) At the end of the process, the particle remains in the state $|g, 0\rangle$, where it can no longer be excited by the laser; thus, the state $|g, 0\rangle$ is a “dark state”. This last condition requires to operate in the *strong confinement limit*, whereby the bandwidth of the excited internal state Γ is smaller than the trap frequency ($\Gamma < \nu$).

Despite the fact that for optical dipole transitions typically $\Gamma > \nu$, one can satisfy the strong confinement condition by either using a metastable excited state and quenching its population using a third (fast decaying) level [Fig. 1(a)], or by using a Raman transition [Fig. 1(b)]. In both cases, given that the extra level is always almost empty, one can eliminate it adiabatically, and obtain an effective two-level transition with a “designed” effective spontaneous rate $\Gamma_{\text{eff}} < \nu$ [15]. The condition related to the LDL, however, cannot be manipulated in a similar way. For $\eta \geq 1$, spontaneous emission will spread the population of the level $|e, n\rangle$ into $|g, n \pm m\rangle$, where $m \sim 0, 1, \dots, \mathcal{O}(\eta^2)$, since the spontaneously emitted photon will typically provide the atom with one recoil $\epsilon_R = \eta^2 \hbar\nu$ of energy. This will produce a diffusing (heating) mechanism that

will prevent most of the atoms from ending up in the ground state $|g, 0\rangle$.

Our proposal overcomes the problem of diffusion, leading to ground state cooling. It consists of driving the atom(s) with two sequences of laser pulses:

(1) The *first* pulse plays a *confinement* role. It drives the atomic population to the levels $|g, n\rangle$ with energy less than one recoil (*i.e.* $n \leq \eta^2$). This is achieved by selecting the laser detunings $\delta \simeq -\hat{\eta}^2\nu$, where $\hat{\eta}^2$ denotes the closest integer $\geq \eta^2$. The effect of these pulses is similar to the traditional sideband cooling: (i) the laser induces transitions $|g, n + \mathcal{O}(\eta^2)\rangle \rightarrow |e, n\rangle$, whereas spontaneous emission produces transitions $|e, n\rangle \rightarrow |g, n + m\rangle$, with $-\eta^2 \leq m \leq \eta^2$, and therefore some energy is in average lost in each absorption–emission cycle; (ii) At the end of the pulse sequence, all the population remains in the states $|g, n\rangle$ with $n \leq \eta^2$, since they are dark with respect to the laser.

(2) The second pulse sequence pumps atoms into the ground state. This is achieved by emptying the motional levels ($|g, 1\rangle, |g, 2\rangle, \dots$) while leaving the population of the ground level $|g, 0\rangle$ basically untouched. This purpose can be realized (*i*) through an appropriate selection of the pulses frequency. In each cycle, some of these atoms will fall in the ground level $|g, 0\rangle$, while the ones heated will be then reconfined after a repetition of the first sequence of pulses. An important ingredient of this scheme is that beyond the LDL, these lasers have to be *blue detuned*. (*ii*) Another realization of the emptying stage can be achieved through the application of Rabi pulses which are multiples of 2π cycles for the ground state $|g, 0\rangle$, so that $|g, 0\rangle$ is a trapping state [16]. This requires that the pulses are sufficiently short so that incoherent processes can be neglected. From now on we will discuss the (*i*) emptying scheme, for which a rate equation treatment is suitable.

Let us start considering a two level internal transition. Assuming $\Omega \ll \Gamma$, where Ω is the Rabi frequency and Γ the excited state decay rate, and $\nu \gg \Omega^2/\Gamma$, one can eliminate the internal excited level, and obtain a set of rate equations for the populations $P_n^g = \langle g, n | \rho | g, n \rangle$:

$$\dot{P}_n^g = - \left[\sum_{m=0}^{\infty} \Gamma_{m \leftarrow n} \right] P_n^g + \sum_{m=0}^{\infty} \Gamma_{n \leftarrow m} P_m^g. \quad (1)$$

According to Eq. (1) the population of level $|g, n\rangle$, increases (decreases) due to transitions from (to) the levels $|g, m\rangle$ at a rate $\Gamma_{n \leftarrow m}$ ($\Gamma_{m \leftarrow n}$). This rate is given by

$$\Gamma_{n \leftarrow m} = \frac{\Omega^2}{\Gamma} \int_{-1}^1 du N(u) \left| \sum_{k=0}^{\infty} \frac{\gamma \langle n | e^{-ik_L x} | k \rangle \langle k | e^{ik_L x} | m \rangle}{\delta - \nu(k - m) + i\gamma} \right|^2, \quad (2)$$

where k_L is the laser-field wavevector and $N(u)$ angular distribution of the spontaneously emitted photon. The rate at which level $|n\rangle$ is emptied is given by

$$\Gamma_n = \sum_{m \neq n} \Gamma_{m \leftarrow n} \simeq \frac{\Omega^2}{\Gamma} \sum_k \frac{\gamma^2 |\langle k | e^{ik_L x} | n \rangle|^2}{[\delta - \nu(k - n)]^2 + \gamma^2}. \quad (3)$$

In the strong confinement limit ($\gamma \ll \nu$), and for a laser frequency such that $\delta = -\nu k_0$ (for certain integer number k_0), we can neglect the non-resonant terms to obtain

$$\Gamma_n \simeq \begin{cases} \frac{\Omega^2}{\Gamma} |\langle n - k_0 | e^{ik_L x} | n \rangle|^2 & \text{if } n > k_0, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

In Fig. 2 we have plotted the population P_0 as a function of the Lamb-Dicke parameter η , after 200 cycles for different detunings. The curves have been obtained by solving numerically the rate equations (1) with a truncated set of states, using the rates (2), where the initial

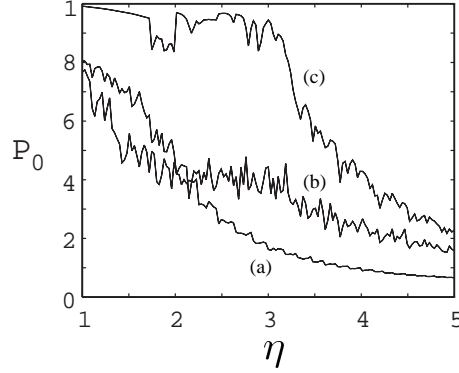


Fig. 2. – Population in $|g, 0\rangle$ as a function of η after 200 sequences of pulses, for $\Gamma = 0.1\nu$, and each pulse of duration $t = 0.1\nu^{-1}$. The curve (a) represents a sequence of one single pulse detuned with $\delta = -\max(2, (\hat{\eta}^2))\nu$; (b) represents a sequence of one single pulse with $\delta = -\nu$; (c) represents a sequence of pulses respectively with $\delta_1 = -\max(2, (\hat{\eta}^2))\nu$ and $\delta_2 = -\nu$.

distribution was taken to be a thermal one with mean $\langle n \rangle = 6$. Curves (a) and (b) correspond to cycles with a single laser pulse of detuning $\delta_{a,b} = -\nu$ and $-\max(2, \hat{\eta}^2)\nu$, respectively, where $\hat{\eta}^2$ denotes the closest integer $\geq \eta^2$. Curve (c) on the other hand corresponds to cycles composed of two different laser pulses of detunings $\delta_{a,b}$, respectively. As this figure shows, with this last scheme one has more than 90% of the atoms in the ground state, even for value of η up to 3. In fact, according to (4), the first laser pulse confines the atoms in the levels $|n\rangle$ with $n < \eta^2$, which are approximately dark with respect to this laser configuration [17], provided that the pulse durations are $t \sim \Gamma_{n \approx \hat{\eta}^2}$. Furthermore, the population in $|n\rangle$ with $n \geq \hat{\eta}^2$ will be distributed among states $|m\rangle$ with $n - 2\eta^2 \leq m \leq n$ in each laser absorption-spontaneous emission cycle [18]. The accumulation of atoms in $|g, 0\rangle$ is achieved by using a laser pulse tuned to the lower motional sideband, as in standard sideband cooling.

For $\eta \geq 3$, however, the efficiency of the method drops drastically. The reason for this behaviour lies in the form of the Franck-Condon coefficient appearing in (4), since it becomes exponentially small for large η and $k_0 \simeq 0$. Therefore, the laser pulse tuned to the lower motional sideband loses its effectiveness, and one has to find another way to empty all the levels different from $|g, 0\rangle$. This can be achieved by using *blue detuned* pulses, provided they are chosen in such a way that $\langle k_0 | e^{ik_L x} | 0 \rangle$ (i.e. Γ_0) is small, but $\langle k_0 + 1 | e^{ik_L x} | 1 \rangle$ (i.e. Γ_1) is sufficiently large, as explained below. We show in Fig. 3 the final populations P_n after 200 cycles of laser pulses, for $\eta = 5$. In Fig. 3(a) we have simulated numerically cycles with a single laser pulse of duration $t_1 = 0.6\Gamma/\Omega^2$ and detuning $\delta_1 = -24\nu$ (i.e. $\delta_1 \simeq -\mathcal{O}(\eta^2)\nu$). In Fig. 3(b) we have simulated cycles consisting of 4 laser pulses of durations $t_{1,2} = 0.6\Gamma/\Omega^2$, $t_{3,4} = 0.2\Gamma/\Omega^2$, and detunings $\delta_{1,2,3,4} = -24\nu, -25\nu, +7\nu, +9\nu$, respectively. As the figure shows, in this last case one can obtain about a 90% of the population in the ground state of the harmonic potential. Note that by choosing a blue detuned pulse with $\delta = +7\nu$ and a pulse duration such that $\Gamma_0 t \ll 1$ the excitation of the state $|g, 0\rangle$ is negligible. This pulse, however still empties states $|g, n\rangle$ ($n \geq 1$), since $\Gamma_0/\Gamma_1 \ll 1$ as it is displayed in Fig. 4, where the rates Γ_n are plotted in function of n . Note that, due to the oscillatory nature of the Franck-Condon coefficient, some states with $n \neq 0$ can be negligibly coupled to the laser field during both the confining pulses and the emptying pulses, leading to a loss of effectiveness of the cooling

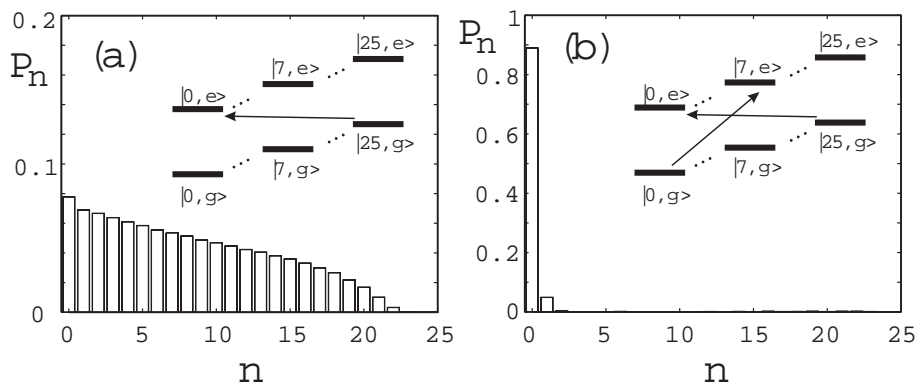


Fig. 3. – Probability distributions P_n for $\Gamma = 0.1\nu$ and $\eta = 5$ after 200 cycles, starting from a thermal state with a mean number 6. Each cycle is composed of: (a) single laser pulse of duration $t_1 = 0.6(\Gamma/\Omega^2)$ and detuning $\delta_1 = -24\nu$; (b) four laser pulses of duration $t_1 = t_2 = 0.6(\Gamma/\Omega^2)$ and $t_3 = t_4 = 0.2(\Gamma/\Omega^2)$, and detunings $\delta_{1,2,3,4} = -24\nu, -25\nu, 7\nu$, and 5ν .

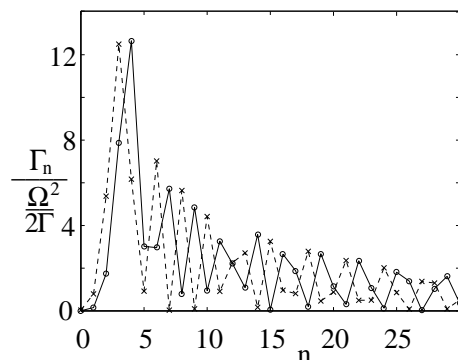


Fig. 4. – Γ_n (in units of Ω^2/Γ) as a function of the Fock number n , for $\eta = 5$. The curve with ‘o’ corresponds to $\delta = 7\nu$ and the one with ‘x’ to $\delta = 9\nu$. In both cases the ratio $\Gamma_0/\Gamma_1 \simeq 0.04$.

mechanism. This problem can be overcome by applying, while confining the atoms, a second laser pulse with $\delta = -(1 + \eta^2)\nu$, and while emptying a level, another blue detuned pulse that satisfies the requirements cited above. This is precisely the role of the pulses with detunings $\delta_{2,4}$ of Fig. 3. Finally, it is worth mentioning that the cooling time scale is independent of the initial temperature of the atoms, when the condition $\langle n \rangle \leq \mathcal{O}(\eta^2)$ is fulfilled, *i.e.* when the initial atoms temperature is below the recoil limit.

Calculations performed for the cooling scheme based on trapping states yield very similar results where typically more than 90% of the atoms are pumped to the ground state.

In most of the practical situations, the strong confinement condition $\Gamma < \nu$ has to be achieved either by using a Raman transition or by quenching a metastable level [see Figs. 1(a,b)], as in laser cooling in the LDL. On the other hand, we have used here a 1-dimensional model. Whereas this is the situation in a linear ion trap, it is not so in

Bose–Einstein condensation experiments. In that case, one should alternate pulses along different directions, as one does in dark state cooling [11].

In conclusion, we have proposed here a novel laser cooling scheme for trapped particles that operates in the strong confinement limit and is not restricted to the LDL, that makes possible the cooling of the atoms into the ground state of the harmonic confining potential. This scheme may be useful in quantum computing as well as Bose–Einstein condensation.

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